

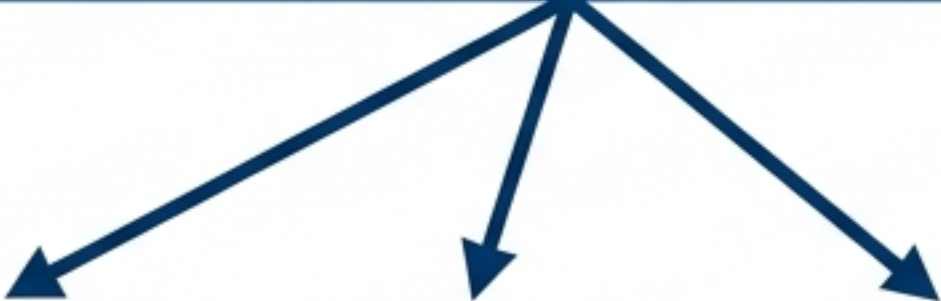


D3h Symmetry as a Geometric Origin of the Koide Formula

**Based on paper P57: The Mathematical Derivation Arc
Focus: Three Fermion Generations & Substrate Parametrization**

An Unexplained Empirical Axiom of Anomalous Precision

PDG 2022 Inputs: 0.510999 MeV, 105.6584 MeV, 1776.86 MeV


$$Q = \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = 2/3$$

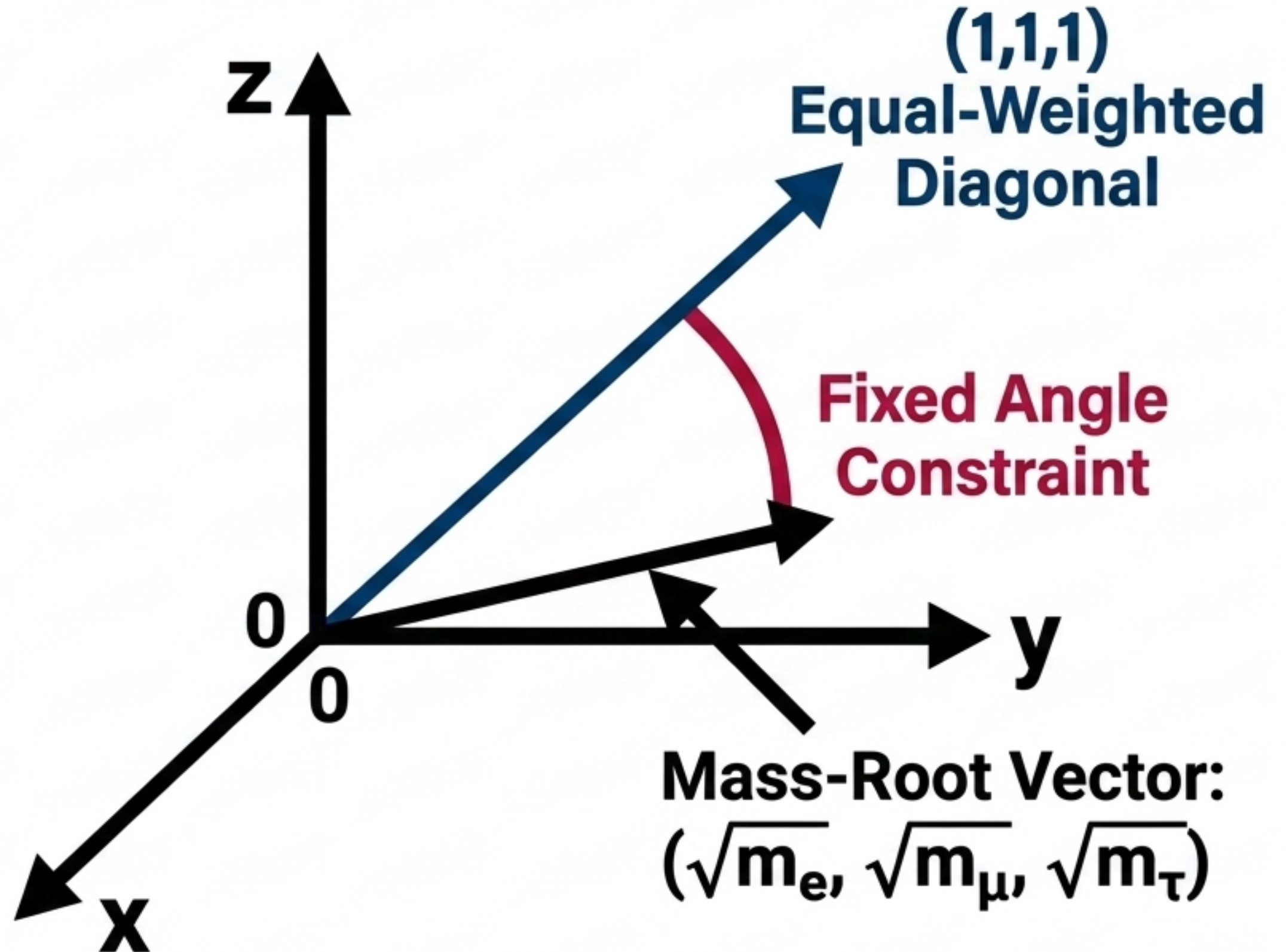


Measured: 0.666661 | Exact Fraction: 0.666667 | Agreement: 0.001%

For over four decades, this precise numerical relation among charged lepton masses has lacked a theoretical derivation from first principles.

Translating a Numerical Coincidence into a Geometric Constraint

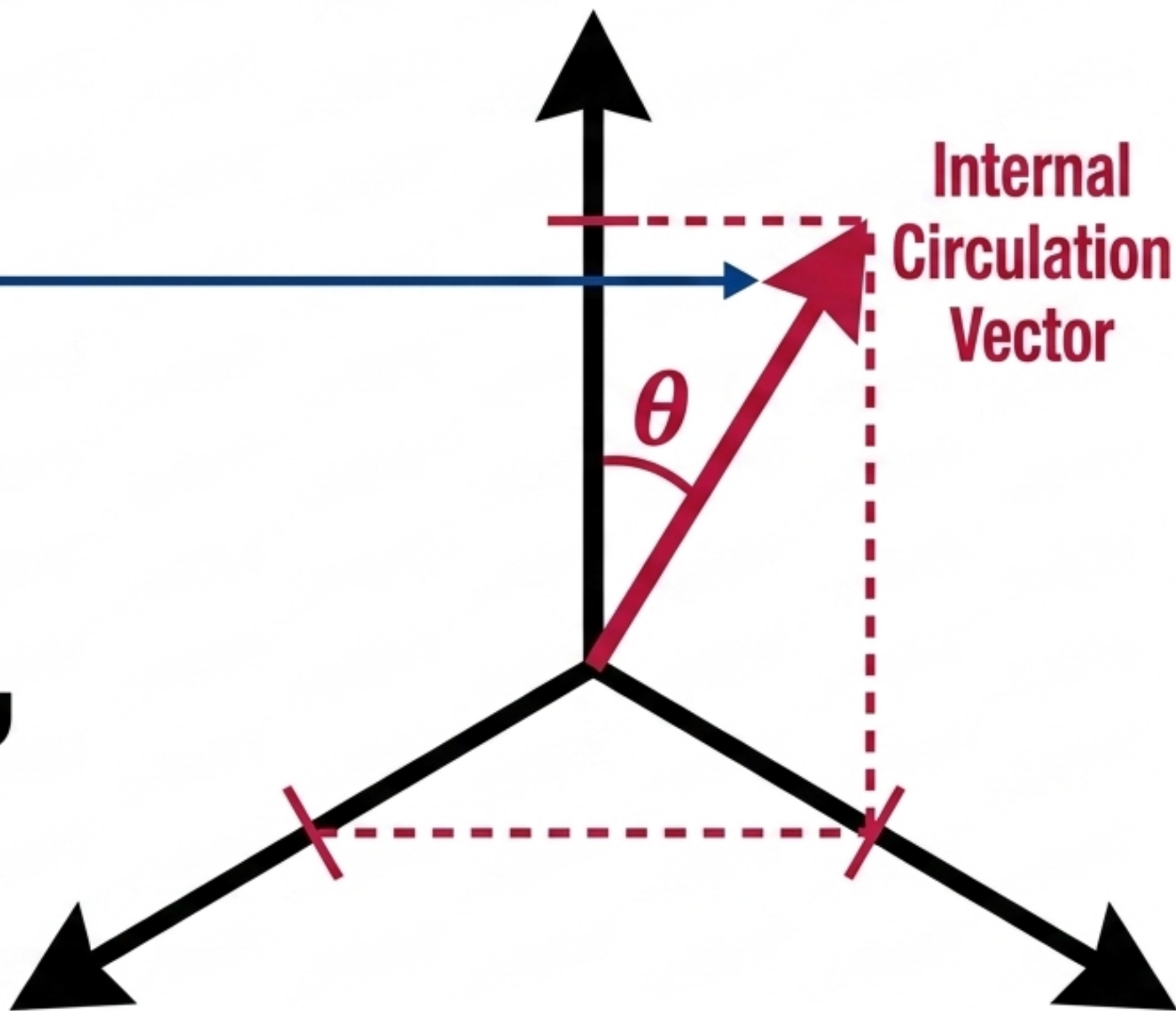
The numerical ratio is geometrically equivalent to a vector of square-root masses maintaining a strictly fixed angle with the equal-weighted diagonal in 3D space.



Deriving Mass from D_{3h} Three-Fold Projection

$$\sqrt{m_n} \propto 1 + \sqrt{2} \cos\left(\theta + \frac{2\pi n}{3}\right),$$

for $n = 0, 1, 2$



Three generation masses emerge not as independent parameters, but as three geometric projections of a single internal circulation vector onto axes fixed by D_{3h} rotational symmetry.

Fixing the Phase Parameter via Substrate Mode Counting

$$\theta = (2\pi + Q) / 3 \text{ (with } Q = 2/3\text{)}$$



Structural Offset: The fraction of the total reconfiguration budget carried by a single mode of the three-core structure.

Fundamental Domain: One full rotation divided among three circulation axes.

The phase θ is completely constrained. The mode-counting fraction $Q = 2/3$ is derived from the substrate's reconfiguration energy independently of any lepton mass data.

Defining the Overall Koide Mass Scale

Absolute Mass Anchor

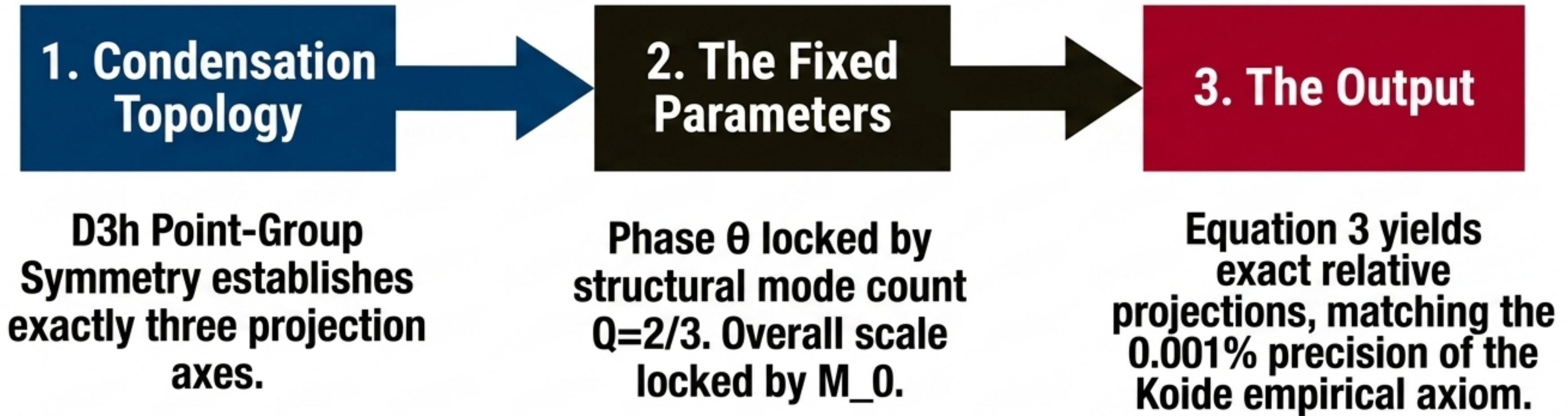
$$M_0^2 = \frac{m_e + m_\mu + m_\tau}{Q}$$

$$M_0^2 = \frac{m_W}{256}$$

Bifurcation Threshold (4^4):
Linking the charged lepton scale structurally to the W boson.

The single phase parameter defines the hierarchy. The absolute scale is anchored structurally to the electroweak sector, requiring zero free parameters to predict the muon and tau masses.

A Closed Mathematical System with Zero Free Parameters

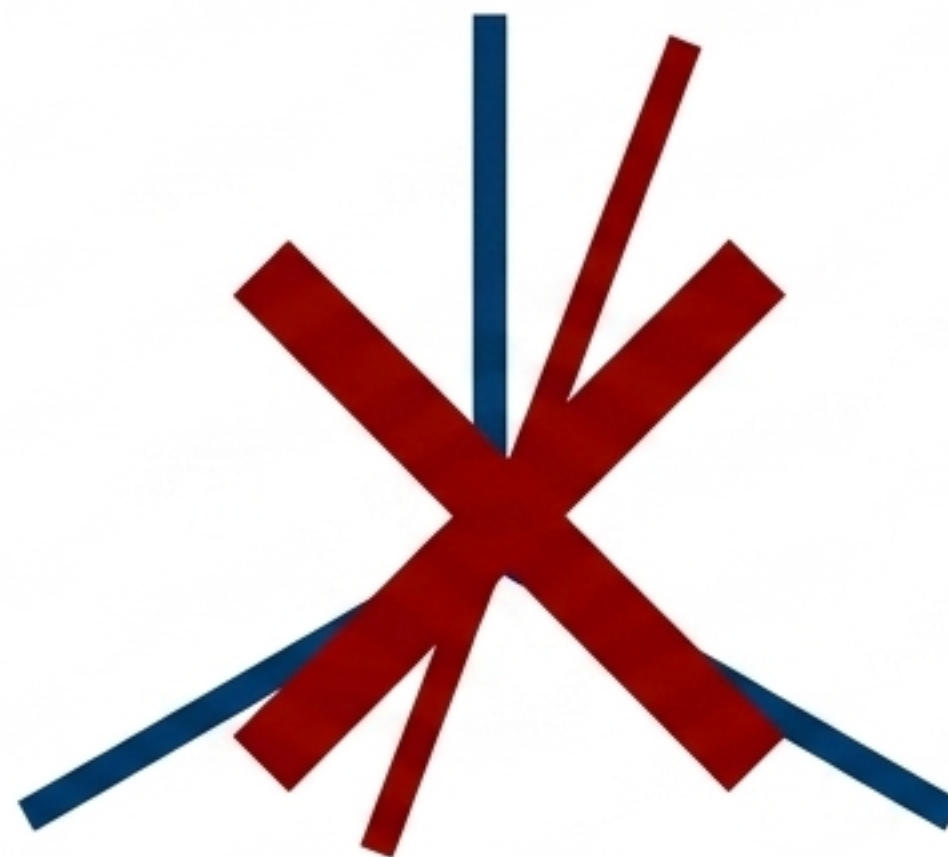


The masses are not independent Standard Model parameters. They are three inevitable geometric projections of a single underlying substrate structure.

The Geometric Limit: Why Exactly Three Generations Exist



Stable 3-Fold Circulation.



A hypothetical 4th generation ($n=4$) requires either a 4th projection axis—destroying the D_{3h} symmetry of the underlying core—or an excitation falling below the required substrate coherence threshold.

The condensation hierarchy admits exactly three stable circulation projection axes. No fourth stable projection exists within this topological class.